بسم الله الرحمن الرحيم

In the name of God

Caspian Journal of Computational & Mathematical Engineering (CJCME)

Director & Licensee : Dr. Seyed Mohammadali Raeisian
Editor in Chief : Dr. S.M.Ali Raeisian
License No : 75729
ISSN (Print) : 2476 – 4418 ,
ISSN (Online) : 2476 – 5252

Email address : CJCMEmail@gmail.com
Postal address : P.O .Box 44815 -1165 (Dr.S.M.Ali Raeisian), Roodsar, Gilan , Iran
Phone : (+98) 13-42 613646 , (+98) 911 143 4988
Website : https://caspianjcme.com/

Editing & Paging : Rah-poyan Andisheh & Honar cultural & artistic institute.
rpandishehonar@gmail.com

Price (Inside of Iran) : 200 000 IRR
Price (Outside of Iran + Shipping cost) : 20 $

September 2017 , No.2

Copyright © 2017 CJCME
All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the written permission of the copyright owner.

Submission Guidelines

Manuscript

- Paper must be current and their main contribution have not been published elsewhere .
- The title of the paper must concise and informative . It is highly desirable to divide the paper into different sections , such as Introduction , Problem Statements , Results , Discussion , and Conclusion .
- Presented papers must include an abstract of maximum 70 words .
- Your article(s) must be classified with MSC 2010 .
- The manuscript must be prepared and submitted as a Word file *(Maximum 20 pages)* .
- The name(s) , affiliation(s) , address(es) , and email(s) must be included .
- You must submit the paper electronically to journal email address *CJCMEmail@gmail.com*

Review process

- Article should be submitted to the related *CJCME* reviews team and undergo rigorous peer review .
- Your name , address , and email will be seen by reviewers .

- The journal has permission to accept , reject , remove , or summarize some part of the manuscript for publication .

Copy right

Knowing the fact that the journal publishes results of academic and research activities of faculty members and researchers in all engineering and fundamental sciences fields free of any publication fee , papers are considered on the understanding that , if they are accepted for publication , the entire copy right shall pass to CJCME journal . All authors are asked to sign a copy right agreement to this effect . The completed form with your final revised paper must be submitted . Any paper published in this journal may be republished in print or in another online venue .

Contents

Numerical Analysis of nanofluids with convective heat transfer through porous disks , Muhammad Fiaz Hussain, Zubair Akbar

…………………………………………………………………………………… P.5

Common Region between Associative Algebras and Non-Associative Lie Algebras , Mehsin Jabel Atteya

…………………………………………………………………………… P.27

Numerical Analysis of nanofluids with convective heat transfer through porous disks

Muhammad Fiaz Hussain,
Department of Mathematics, COMSATS Institute of Technology, 57000, Sahiwal, Pakistan
Fiaz.hussain24@yahoo.com

Zubair Akbar
Centre for Advanced Studies in Pure and Applied Mathematics, Bahauddin Zakariya University, Multan, Pakistan
akbar_zubair@yahoo.com

AMS 2010 Mathematical Subject Classification : 76E15 , 76F35, 76S05

***Keywords*:** Nanofluids, Convective Boundaries, Porous Disks.

DOI : http://dx.doi.org/10.22039/cjcme.2017.05

Abstract: We constructed a mathematical model for analyzing convection heat and mass transfer of an incompressible viscous nanofluid through porous disks in the presence of metallic nanoparticles for different problems. We compare the results by adding the convective boundaries. Governing equations are convert into non-linear ordinary differential equations. Numerically we solve by FD discretization (SOR). The influence of different parameters on the heat and mass transfer profiles are shown.

1. INTRODUCTION

The twenty first century is an era of technological development and has already seen many changes in almost every industry. The introduction of nano science and nanotechnology by the Nobel Prize winning physicist Richard Feynman in 1959. Feynman proposed the concept of micro machines. In 1974 Scientist Norio Taniguchi first used the term “Nanotechnology”.

Began more than a century ago, the great scientist James Clark Maxwell [1] develop a theoretical model of the electrical conductivity of solid particles. Since then, the classical Maxwell model has been applied while investigation the thermal conductivity of mixtures of solid particles and liquids. However, all these studies have been conducted with millimeter or micrometer sized particles. The major problem with the use of micro particles is that they settle very rapidly in liquids. They also cause abrasion, clogging, and additional pressure drops. Furthermore, high particles concentrations are required to obtain appreciable improvements in the thermal conductivities of these suspensions. Nanofluids are new class of nanotechnology-based heat transfer fluids, obtained by dispersing and stably suspending nanoparticles with typical dimensions on the order of 1-10 *nm*. Many talented scientists in the rapidly growing nanofluids community have made important scientific discoveries not only in the discovering unexpected thermal properties of nanofluids, developing new mathematical models for the nanofluids, identifying the unusual opportunities to develop next generation coolants such as a smart coolants for computers and safe coolants for nuclear reactors. After that Nanofluids (nanoparticles fluids suspensions) coined by Choi [2] in to describe the new class of nanotechnology-based heat transfer fluids with augmented thermal properties, both superior to the properties of their own hosting fluids and conventional fluid suspensions. The goal of nanofluids is to achieve the highest possible thermal properties at the smallest possible concentrations (preferably $<1\%$ by volume) by uniform dispersion and stable suspension of nanoparticles (preferably $<10\ nm$) in host fluids. To achieve this goal it is vital to understand how nanoparticles enhance energy transport in liquids. As a result, the research regarding nanofluids is now receiving increasing interest worldwide, as shown by the exponentially increasing number of publications.

Transport in porous medium channel geometry has broad applications in the biological, physical and chemical parts for transportation of broad amounts, for example, the mass of a liquid, mass of a segment of a phase, seepage, energy and momentum, in single and multiphase flow in porous media realm. In mechanical procedures, the porous medium is used for enhancing the convection heat and mass transfer properties. [3] Convection fluid flow, heat and mass transfer with a porous medium happen in power station of numerous design characteristics where cooling or heating is required, for example, cooling turbine sharp blades, cooling electronic hardware, filtration, warm protection, ground water, oil stream and a wide range of heat exchangers. As per some fundamental focal points of utilizing a porous medium, first its dissipation region is larger than the usual fins that increase the heat convection. Another point is the irregular movement of the fluid flow around the individual globules which blends the fluid all the more viably. That's why; it would be the best combination; porous medium and nanofluid, for effective convective heat transfer.

Flow and heat transfer of nanofluids over a rotating disk with uniform stretching rate in the radial direction represented by Yin [4]. Naveed [5] recently work on convective heat transfer of Al_2O_3 nanofluids in porous media on MHD flow of nanofluids due to a rotating disk with slip effect. A numerical study working by Hayyat [6] also the numerical investigation on heat absorption worked by Anjali and Priya [7]. Mehdi et al. [8] presented the investigation of the magnetic field has critical reasonable applications in medical and engineering sciences. Numerous mechanical sorts of supplies, for example, pumps, bearing, MHD generators, and boundary layer flow control are influenced by the connection between a magnetic field and electrically conducting fluid. Sheikholeslami et al. [9] analyzed the effect of magnetic field in the nanofluid flow in a porous channel. Fakour et al. [10] showed an analytical solution for micro polar fluid flow through a channel with porous walls. Jashim et al. [11] demonstrated the mathematical modeling of MHD thermo solute nanofluid flow in porous medium under the influence of convection slip conditions. Hayat et al. [12] presented an analysis of heat transfer on the MHD peristaltic flow with porous medium. Sacheti [13] discussed the Brinkman model for the steady poisue flow of a viscous incompressible fluid in a porous channel. Various researchers have all that much purposeful this idea and the points of interest can be found in writing.

To our best knowledge no researcher has yet explored the mass and heat transfer characteristics in nanofluid between two orthogonally moving porous disks with convection of boundaries. Therefore, in this paper, we consider the combined effect of heat and mass transfer in unsteady laminar incompressible flow of a nanofluid between two orthogonally moving porous coaxial disks. A similarity transformation is used to reduce the governing partial differential equations to a set of nonlinear coupled ordinary differential equations in the dimensionless form, which are numerically solved by employing an algorithm based on the Quasi-linearization & finite difference discretization. The ease in obtaining the numerical solution using the technique makes it superior than the shooting like approach used in our earlier investigations. The effects of the governing parameters on the flow and heat transfer aspects of the problem are discussed.

2. PROBLEM STATEMENT

We consider unsteady laminar viscous flow and heat transfer of an incompressible nanofluid (containing TiO_2 nanoparticles) between two orthogonally moving porous coaxial disks with suction and all the thermo physical properties are assumed to be constant. The base fluid is taken to be water which is in thermal equilibrium with the nanoparticles, with no slip occurring between them. The disks have the same permeability, move down or up uniformly at a time dependent rate $a'(t)$, and are at a distance $2a'(t)$ apart.

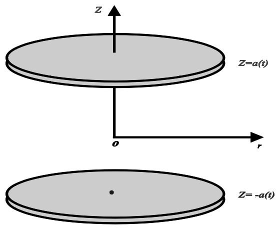

FIG. 1: Physical model of the problem.

The geometry of the problem (shown in Figure 1) suggests that the cylindrical coordinate system may be chosen with the origin at the middle of the two disks. With *u* and *w* being the velocity components in, respectively, the r- and z- directions, the governing equations for the problem, taking the viscous dissipation effect into account are,

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r}+\nu_{nf}\left(\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial z^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right), \tag{2}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial z}+\nu_{nf}\left(\frac{\partial^2 w}{\partial r^2}+\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right), \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha_{nf}\frac{\partial^2 T}{\partial z^2}+\frac{\mu_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial u}{\partial z}\right)^2, \tag{4}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}=D\nabla^2 C, \tag{5}$$

Where C is the concentration, p is the pressure, and T is the temperature. α_{nf} is the thermal diffusivity, ρ_{nf} is the density, υ_{nf} is the kinematics viscosity of the nanofluid, which are given by,

$$\begin{gathered}\nu_{nf}=\frac{\mu_{nf}}{\rho_{nf}},\ \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\ \rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\ \alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}},\\ (\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s,\ \frac{k_{nf}}{k_f}=\frac{(k_f+2k_s)-2\phi(k_f-k_s)}{(k_f+2k_s)+\phi(k_f-k_s)},\end{gathered} \tag{6}$$

where ρ_f and ρ_s are, respectively, the densities of the solid fractions of the fluid, $(\rho cp)_{nf}$ is the heat capacitance of the nanofluid, k_{nf} is the effective thermal conductivity of the nanofluid.

The boundary conditions are,

$$\begin{gathered}u=0,\ w=-Aa'(t),\ -k_{nf}\frac{\partial T}{\partial z}=h_f(T_1-T),\ C=C_1 \text{ at } z=-a(t),\\ u=0,\ w=Aa'(t),\ T=T_2,\ C=C_2 \text{ at } z=a(t),\end{gathered} \tag{7}$$

Here A is the measure of the wall permeability, h_f denotes the heat transfer and the dash denotes the derivative w. r. t. the time t. After eliminating the pressure term from the governing equations, We introduce the following similarity transformations,

$$\eta=\frac{z}{a},\ u=-\frac{r\nu_f}{a^2}F_\eta(\eta,t),\ w=\frac{2\nu_f}{a}F(\eta,t),\ \theta(\eta)=\frac{T-T_2}{T_1-T_2},\ \chi(\eta)=\frac{C-C_2}{C_1-C_2}, \tag{8}$$

After simplify the above equations we arrived at,

$$\frac{\nu_{nf}}{\nu_f}F_{\eta\eta\eta\eta}+\alpha(\eta F_{\eta\eta\eta}+3F_{\eta\eta})-2FF_{\eta\eta\eta}-\frac{a^2}{\nu_f}F_{\eta\eta t}=0, \tag{9}$$

$$\theta_{\eta\eta}-\frac{\nu_f}{\alpha_{nf}}(2F-\eta\alpha)\theta_\eta+(1-\phi)^{-2.5}F_{\eta\eta}^2 Ec\,\Pr\left(\frac{k_f}{k_{nf}}\right)-\frac{a^2}{\alpha_{nf}}\theta_t=0, \tag{10}$$

$$D\chi_{\eta\eta}=a^2\chi_t+\nu_f\chi_\eta(2F-\eta\alpha), \tag{11}$$

with boundary conditions,

$$\begin{aligned} F=-\mathrm{Re},\ F_\eta=0,\ \theta_\eta=Bi(1-\theta),\ \chi=1,\ \eta=-1,\\ F=\mathrm{Re},\ F_\eta=0,\ \theta=0,\ \chi=0,\ \eta=1, \end{aligned} \tag{12}$$

Here T_1 and T_2 (with $T_1> T_2$) are the fixed temperatures and C_1 and C_2 are the fixed concentrations of the lower and upper disks respectively, $\alpha=\frac{aa\prime(t)}{v_f}$ is the wall expansion ratio, $Re=\frac{Aa a\prime(t)}{2v_f}$ is the permeability Reynolds number, $Pr=\frac{(\mu c_p)_f}{k_f}$ is the Prandtl number, $Bi=\frac{ah_{nf}}{k_{nf}}$ is the Biot number, $Sc=\frac{v_f}{D}$ is the Schmidt number and $Sc=\frac{U_2}{(T_1-T_2)(cp)_f}$ is the Eckert number, where $U=\frac{rv_f}{a^2}$ be the reference velocity.

It is worthy to note that the continuity Eq. (1) is identically satisfied which means that the proposed velocity is compatible with (1) and, therefore, represents the possible fluid motion.

Finally, we set $f=\frac{F}{Re}$, and consider the case following Majdalani et al. [14] when $\alpha$is a constant, $f=f(\eta),\theta=\theta(\eta)$ and $\chi=\chi(\eta)$, which leads to $F_{\eta\eta t}=0,\theta_t=0$ and $\chi_t=0$. Thus we have the following equations,

$$\frac{\nu_{nf}}{\nu_f}f_{\eta\eta\eta\eta}+\alpha(\eta f_{\eta\eta\eta}+3f_{\eta\eta})-2\,\mathrm{Re}\,ff_{\eta\eta\eta}=0, \tag{13}$$

$$\theta_{\eta\eta}-\frac{\nu_f}{\alpha_{nf}}(2\,\mathrm{Re}\,f-\eta\alpha)\theta_\eta+\mathrm{Re}^2(1-\phi)^{-2.5}f_{\eta\eta}^2 Ec\,\mathrm{Pr}\left(\frac{k_f}{k_{nf}}\right)=0, \tag{14}$$

$$\chi_{\eta\eta}=Sc\chi_\eta(2\,\mathrm{Re}\,f-\eta\alpha), \tag{15}$$

with boundary conditions,

$$\begin{gathered} f=-1,\ f'=0,\ \theta'=Bi(1-\theta),\ \chi=1,\ \eta=-1, \\ f=1,\ f'=0,\ \theta=0,\ \chi=0,\ \eta=1, \end{gathered} \tag{16}$$

3. SOLUTION FOR THE PROBLEMS

An inspection of Eq. (13) reveals that it can be readily integrated & becomes,

$$\frac{\nu_{nf}}{\nu_f}f_{\eta\eta\eta}+f_{\eta\eta}(\eta\alpha-2\,\mathrm{Re}\,f)+f_\eta(2\alpha+\mathrm{Re}\,f_\eta)=\beta, \tag{17}$$

where β is the constant of integration to be determined.

We consider this third order equation as the following system of coupled 1^{st} order & 2^{nd} order ODEs,

$$f_{\eta} = q, \tag{18}$$

$$\frac{\nu_{nf}}{\nu_{f}} q_{\eta\eta} + q_{\eta}(\eta\alpha - 2\,\mathrm{Re}\, f) + q(2\alpha + \mathrm{Re}\, q) = \beta, \tag{19}$$

We solve the above equations along with Eqs. (14), subject to the boundary conditions,

$$f(\pm 1) = \pm 1,\ q(\pm 1) = 0,\ \theta(-1) = 1,\ \theta(1) = 0,\ \chi(-1) = 1,\ \chi(1) = 0 \tag{20}$$

For the numerical solution of the present problem, we first discretize Eqs. (14) And (19) at a typical grid point $\eta = \eta_i$ by employing central difference approximations for the derivatives. The resultant algebraic system is solved iteratively by SOR method, subject to the appropriate boundary conditions given in Eq. (20). On the other hand, Eq. (18) is integrated numerically by employing the fourth order Runge Kutta method, after every SOR iteration.

It is important to mention that the boundary condition $f(1) = 1$ is not needed for the above mentioned computational procedure, and can be used in finding the constant of integration β which is the only unknown as compared to three missing initial conditions in the usual shooting approach. In our earlier work, we used trial and error method to find β but someone dimensional zero finding algorithms may also be employed for the purpose. However, the sensitivity of β with respect to the governing parameters makes it difficult to find β and some manual effort is always required in every simulation to obtain its desired value. That is why we need some alternative approach which does not require finding any unknown and must entirely base on the FDM. In this paper, we discuss an alternative approach based on the Quasi-linearization of the nonlinear ODEs.

3.1 QUASSI LINEARIZATION METHOD

In quasi-linearization, we construct three sequences of vectors $\{f^{(k)}\}, \{\theta^{(k)}\}$ and $\{\chi^{(k)}\}$ which converge to the numerical solutions of Eqs. (13), (14), and (15),

respectively. To construct $\{f^{(k)}\}$ we linearize Eq. (13), by retaining only the first order terms, as follows:
We set,

$$G(f, f_\eta, f_{\eta\eta}, f_{\eta\eta\eta}, f_{\eta\eta\eta\eta}) \equiv \frac{\nu_{nf}}{\nu_f} f_{\eta\eta\eta\eta} + \alpha(\eta f_{\eta\eta\eta} + 3 f_{\eta\eta}) - 2\,\mathrm{Re}\, f f_{\eta\eta\eta}, \tag{21}$$

$$G\left(f^{(k)}, f_\eta^{(k)}, f_{\eta\eta}^{(k)}, f_{\eta\eta\eta}^{(k)}, f_{\eta\eta\eta\eta}^{(k)}\right) + \left(f^{(k+1)} - f^{(k)}\right)\frac{\partial G}{f^{(k)}} + \left(f_\eta^{(k+1)} - f_\eta^{(k)}\right)\frac{\partial G}{f_\eta^{(k)}}$$
$$+\left(f_{\eta\eta}^{(k+1)} - f_{\eta\eta}^{(k)}\right)\frac{\partial G}{f_{\eta\eta}^{(k)}} + \left(f_{\eta\eta\eta}^{(k+1)} - f_{\eta\eta\eta}^{(k)}\right)\frac{\partial G}{f_{\eta\eta\eta}^{(k)}} + \left(f_{\eta\eta\eta\eta}^{(k+1)} - f_{\eta\eta\eta\eta}^{(k)}\right)\frac{\partial G}{f_{\eta\eta\eta\eta}^{(k)}} = 0, \tag{22}$$

$$\frac{\nu_{nf}}{\nu_f} f_{\eta\eta\eta\eta}^{(k+1)} + \left(\eta\alpha - 2\,\mathrm{Re}\, f^{(k)}\right) f_{\eta\eta\eta}^{(k+1)} + 3\alpha f_{\eta\eta}^{(k+1)}$$
$$-2\,\mathrm{Re}\, f_{\eta\eta\eta}^{(k)} f^{(k+1)} = -2\,\mathrm{Re}\, f_{\eta\eta\eta}^{(k)} f^{(k)}, \tag{23}$$

Now Eq. (23) gives a system of linear differential equations, with $\{f^{(k)}\}$ being the numerical solution vector of the k^{th} equation. To solve the linear ODEs, we replace the derivatives with their central difference approximations, giving rise to the sequence $\{f^{(k)}\}$,

$$Af^{(k+1)} = B \text{ with } A \equiv A\left(f^{(k)}\right) \text{ and } B \equiv B\left(f^{(k)}\right), \tag{24}$$

On the same way, Eq. (14) and (15) are linear in θ and χ, respectively and therefore, in order to generate the sequence $\{\theta^{(k)}\}$ and $\{\chi^{(k)}\}$, these may be written as,

$$\theta_{\eta\eta}^{(k+1)} - \frac{\nu_f}{\alpha_{nf}}(2\,\mathrm{Re}\, f - \eta\alpha)\theta_\eta^{(k+1)} + \mathrm{Re}^2\left((1-\phi)^{-2.5} f_{\eta\eta}^{(k+1)^2}\right) Ec\,\mathrm{Pr}\left(\frac{k_f}{k_{nf}}\right) = 0, \tag{25}$$

$$\chi_{\eta\eta}^{(k+1)} - Sc\chi_\eta^{(k+1)}(2\,\mathrm{Re}\, f^{(k+1)} - \eta\alpha) = 0, \tag{26}$$

It is important to note that $f^{(k+1)}$ is considered to be known in the above equation and its derivatives are approximated by the central differences. We outline the computational procedure as follows,

- Provide the initial guess $\{f^{(0)}\}$, $\{\theta^{(0)}\}$and$\{\chi^{(0)}\}$, satisfying the boundary conditions given in Eq. (20).
- Solve the linear system given by Eq. (23) to find $\{f^{(1)}\}$
- Use $\{f^{(1)}\}$ to solve the linear system arising from the FD discretization of Eq. (25), to get $\{\theta^{(1)}\}$ and $\{\chi^{(1)}\}$.
- Take $\{f^{(1)}\}$, $\{\theta^{(1)}\}$ and $\{\chi^{(1)}\}$ as the new initial guesses and repeat the procedure to generate the sequences $\{f^{(k)}\}, \{\theta^{(k)}\}$ and $\{\chi^{(k)}\}$ which, converge f, θ and χ respectively, (the numerical solutions of Eqs. (13) and Eqs. (14).
- The two success are generated until $\max\{\|f^{(k+1)} - f^{(k)}\|, \|\theta^{(k+1)} - \theta^{(k)}\|, \|\chi^{(k+1)} - \chi^{(k)}\|\} < 10^{-6}$ is reached.

It is important to note that the matrix *A* (in Eq. (24)) is pentadiagonal & not diagonally dominant, and hence the iterative method (like SOR) may fail or work very poorly. Therefore, some direct method like *LU* factorization or Gaussian elimination with full pivoting (to ensure stability) may be employed.

The process of optimization is stopped once the criterion,

$$| \mu_\omega^{(t)} - \mu_\omega^{(t-1)} | < Tol_{\omega opt} \quad (27)$$

is satisfied subject to $t>1$ and $\mu_\omega^{(t)} > 1$. The iterative process is continued with $\omega = \omega opt$ until

Table 1. Water and Ti (titinium nanoparticles) physical properties.

Nanoparticles	$\rho(kgm^{-3})$	$C_p(jkg^{-1}k^{-1})$	$K(wm^{-1}k^{-1})$
H_2O	997.1	4179	0.613
Ti	4250	8.9538	4179

density of the grid points. The coarse grid larger larger values result in a rapid convergence while for at fine grid smaller values may be chosen for better estimation of ωopt.

Table 2. Dimensionless velocity $f(\eta)$ on three grid sizes and extrapolated values for $Re=5$, $\phi = 0.01$, $\alpha=5$, $Ec=0.1$, and $Bi=Sc=0.1$.

η	$f(\eta)$			
	1st grid (h=0.02)	2nd grid (h=0.01)	3rd grid (h=0.005)	Extrapolated values
0.2	0.3182	0.3197	0.3204	0.3207
0.4	0.5979	0.6003	0.6016	0.6027
0.6	0.8131	0.8157	0.8170	0.8174
0.8	0.9499	0.9516	0.9525	0.9528

4. RESULTS AND DISCUSSION

The physical quantities of our interest are the shear stress, heat and mass transfer rates at the disks which are, proportional to $f''(-1), \theta'(-1)$ and $\chi(-1)$ respectivlely. Due to symmetry of the problem, the results are given only at the lower disk. The parameters for the present study are the Reynolds number Re, the nanoparticle volume fraction parameter ϕ, the wall expansion ratio α, the Eckert number Ec, Biot number Bi and the Schmidt number Sc. The traditional way of studying the fluid dynamics problems is to specify the values of these dimensionless groups rather

Than satisfying the particular fluid properties and the domain dimensions. Obviously, the results

Table 3. Effect of the nanoparticle volume fraction parameter ϕ on $f''(-1), \theta'(-1)$ and $\chi(-1)$ for $Re=-5$, $Ec=0.1$, and $Sc=0.1$.

ϕ	$\alpha=5$			$\alpha=-5$		
	$f''(-1)$	$\theta'(-1)$	$\chi'(-1)$	$f''(-1)$	$\theta'(-1)$	$\chi'(-1)$
0.00	1.4514	0.0294	-0.4221	1.6008	0.0949	-0.5899
0.05	1.4484	0.0384	-0.4235	1.600	0.107	-0.5908
0.10	1.4452	0.1508	-0.425	1.5991	0.4317	-0.5927
0.15	1.4418	0.3334	-0.4264	1.5982	0.9799	-0.5946
0.20	1.4382	0.5821	-0.4279	1.5972	1.7568	-0.5965

obtained in this way are applicable to the flow problems with particular values of material properties and the dimensions of the domain, falling in the ranges considered in the present work. It is note that for $\alpha < 0$ or $\alpha > 0$ according to the case when the disks are approaching each other

Table 4. Effect of the wall expansion ratio α on $f''(-1), \theta'(-1)$ and $\chi(-1)$ for *Re*=-5, $\phi = 0.01$, *Ec*=0.1, and *Bi*=*Sc*=0.1.

α	*f''(-1)*	*θ'(-1)*	*χ'(-1)*
-5	-1.6007	5.455	-0.5893
-3	-1.6538	4.2313	-0.5524
-1	-1.7647	3.5374	-0.5041
0	-0.0023	3.2879	-0.5004
1	1.1898	3.0941	-0.4839
3	1.5236	2.8418	-0.4523
5	1.4508	2.7162	-0.4224

or moving away, $\alpha < 0$ or $\alpha > 0$ according to the case when the disks are approaching each other or moving away, whereas $\mathrm{Re} < 0$ for suction .We shall study the effects of the parameters on $f''(-1), \theta'(-1)$ and $\chi(-1)$, as well as, on the two velocity profiles $f(\eta)$ and $f'(\eta)$, the heat profile $\theta(\eta)$and $\chi(\eta)$. The values in (Table 1) show the physical properties of TiO_2 (nanoparticles) and water H_2O (base fluid).

Table 5. Effect of the permeability Reynolds number Re on $f''(-1), \theta'(-1)$ and $\chi(-1)$ for $\phi = 0.01$, *Ec*=0.1, and *Bi*=*Sc*=0.1.

Re	*α=5*			*α=-5*		
	f''(-1)	*θ'(-1)*	*χ'(-1)*	*f''(-1)*	*θ'(-1)*	*χ'(-1)*
-2	0.8248	1.5877	-0.3509	1.7647	2.7592	-0.5346
-1	0.467	0.5551	-0.3848	1.5075	0.82813	-0.5041
0	-0.1589	0.0328	-0.422	1.4289	1.0398	-0.5004
1	-1.3739	1.5066	-0.4641	1.3898	1.2191	-0.4839
2	-3.6456	9.7989	-0.5132	1.3472	3.0542	-0.4679

Table 6. Effect of the Eckert number Ec on $\theta'(-1)$ for *Re=-5, φ=0.01, Pr=2.92,* and *Sc=0.1*.

Ec	*θ'(-1)*	
	α=5	*α=-5*
0	1.2461	1.5675
0.3	4.7629	5.99363
0.5	7.9383	9.98939
0.8	12.701	15.9834
1.1	17.464	21.9767

(Table 2) shows the numerical solution of the given problem which is solved by FDM (SOR) using the MATLAB code. The results concluded that the convergence analysis by increasing the step size. Comparison table constructed with different step size. Also we concluded that the extrapolated values are close to the exact solution.

Table 7. Effect of the Schmidt number *Sc* on $\chi'(-1)$, for *Re=-5*, $\phi = 0.01$, *Pr=2.92,* and *Ec=0.1*.

Sc	$\chi'(-1)$	
	α=5	*α=-5*
0	-0.4794	-0.9797
0.3	-0.7794	-0.6793
0.5	-1.0387	-1.3387
0.8	-1.5232	-1.0232
1.1	-2.1104	-2.1104

The effect of the volume fraction parameter ϕ by adding the nanoparticles *(TiO_2)* with base fluid water. The calculated values on shear stress, heat and mass transfer. The shear stress and mass transfer rate decreased by increasing the volume fraction ϕ. But the heat transfer rate remarkably increased by the increased the volume fraction ϕ for the two cases of α as shown in (Table 3). Also we reached the values of shear stress and mass transfer decrease slightly. The effect of the wall expansion parameter α on the shear stress as well as heat and mass transfer decrease by increasing. Due to symmetry the value of shear stress far away from the center on moving the opposite direction of disks similarly the shear stress come close to the center if the disks are contract to each other. The lower end of the disks for $(\alpha = -1, -2, -3)$ decrease the results of the shear stress but the values increases by increasing the values of wall expansion $(\alpha = 1, 2, 3)$ as shown in (Table 4). This shows that the values of heat and mass transfer increased by increasing the value of α.

It has been observed that the effect of the suction is to reduce the mass transfer rate at the disks whether the disks are approaching to or moving away from each other. On the other hand, the Reynolds number Re significantly reduces the shear stress and the heat transfer rate in case of approaching disks whereas an opposite trend is noted when the disks are moving away as shown in (Table 5).

Table 8. Effect of the Biot number Bi on $\theta'(-1)$, for *Re=-5*, $\phi = 0.01$, *Pr=2.92*, *and Bi=Ec=0.1*.

Bi	$\theta'(-1)$	
	α=5	*α=-5*
0.2	-1.9737	-1.0123
0.4	-1.6735	-1.3793
0.6	-1.3387	-1.5387
0.8	-1.2232	-1.7232
1.0	-1.1104	-2.9104

The Schmidt number Sc decreases the mass transfer rate for both the cases of α , but the decrease is more pronounced in case of receding disks, as shown in the (Table 7). In two cases of α the heat transfer increased large values by increasing the Eckert number Ec (viscous dissipation) in (Table 6).

By the values calculated in the (Table 8) we reached at the stage that the heat transfer rate increased slightly by increasing the Biot number *Bi*. The streamlines for the present problem are shown in Fig. 2 for the fixed values Re = -5, φ =0.1,α = -5. A transition is obvious in the direction of the fluid velocity (that is, from vertical to horizontal) from the disks towards the middle of the domain. It is to mentioned that the position of viscous layer (the point $\eta = \eta_i$ for which $f(\eta) = 0$) lies in the middle of the region between the two disks, that is, in the plane *z* = *0*, due to same amount of suctions at the two disks. However, it may shift towards either of the disks for the asymmetrical case, which may be a topic of a subsequent study. Moreover, the axial velocity takes its dimensionless value -1 at the lower disk and acquires its maximum value1 at the upper one, with a point of inflection lying at $\eta = 0$ (due to the symmetry of the problem) where it changes its concavity. There is a major qualitative difference in the temperature profiles for the approaching and receding disks. The thermal distributions are raised across the whole domain for α < 0 whereas, in the other case, the profiles are more affected only in the middle of the domain

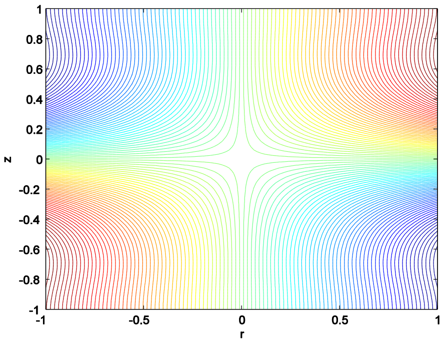

Fig. 2: Contour effect for the problem.

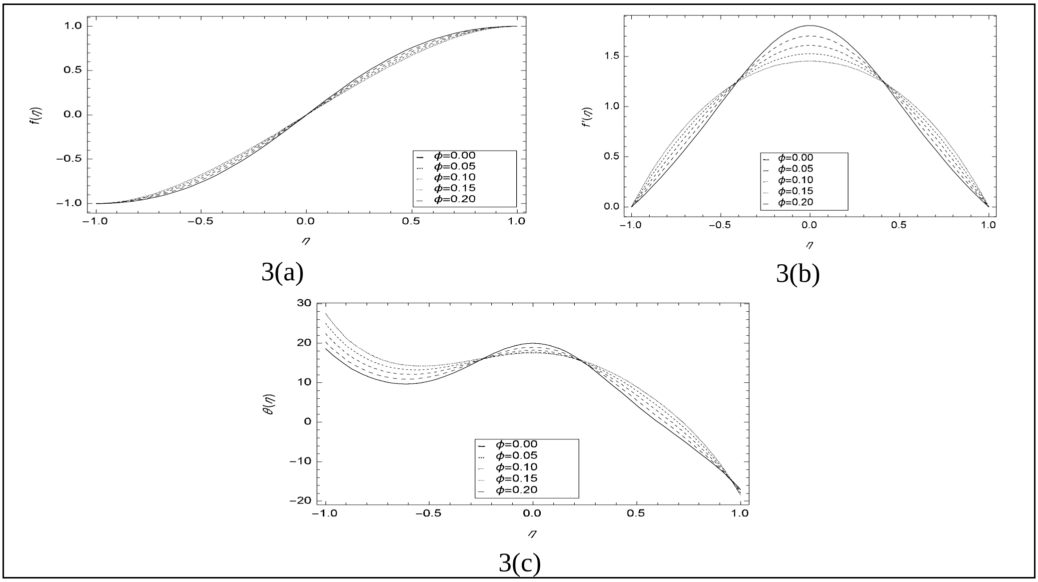

Fig. 3: Effect of the Reynold parameter Re and wall expansion α on velocity profile.

and less influenced near the disks. Similarly, the radial velocity is also noted to be more affected near the disks for the case $\alpha < 0$. As a result the shear stress and the heat transfer rates at the disks acquire higher values for the approaching disks as compared with the receding disks case (as noticed in the tables 2-8).

volume fraction parameter ϕ, for both the cases of α. That is why in the Figs. 3(a) and 3(b), we have given the influence of the parameter ϕ on the axial and radial velocity distribution only, for the fixed values $Re = -1, Sc = 0.1, Ec = 0.1$ and $\alpha = 5$ *or -5*. Moreover, the temperature initial boundary distribution apart the disks, (and hence the heat transfer rate) is not affected pronouncedly by the parameter ϕ for $\alpha > 0$ whereas the thermal distribution is raised across the entire domain when the disks are approaching Fig. 3(c).

For the fixed $\phi = 0.1$, $Sc = 0.1$ and $Bi{=}Ec = 0.1$, the influence of the Reynolds number Re and expanding contracting parameter α on the axial and radial velocity shown Fig. 4(a), 4(b), 4(c) and 4(d). Whereas the concentration profiles are raised only in the lower half of the plane ($z = 0$). For ($\alpha < 0$) Fig. 6(a), both axial and the maximum value of the radial velocity decrease with the Reynolds number. On the other hand, the temperature distribution Fig 5(a) increases across the entire domain. it has been noted that the velocity profiles are affected by Re in an opposite manner as in case of receding disks, but the influence is the same for the concentration profiles. The Reynolds number Re tends to eliminate the flattened nature of the temperature profile by lower it near the disks and significantly raising in the middle of the domain. The influence of the wall expansion ratio α on the velocity, temperature and concentration profiles is given in the (Figures 4-6) for the fixed $\phi = 0.1$, $Ec = 0.1$, $Bi{=}0.1$, $Sc = 0.5$ and $Re = -1$. As α varies from negative to positive, the magnitude of the axial velocity increases on the entire domain, whereas the radial velocity increases only in the middle of the disks while decreasing near the disks. On the other

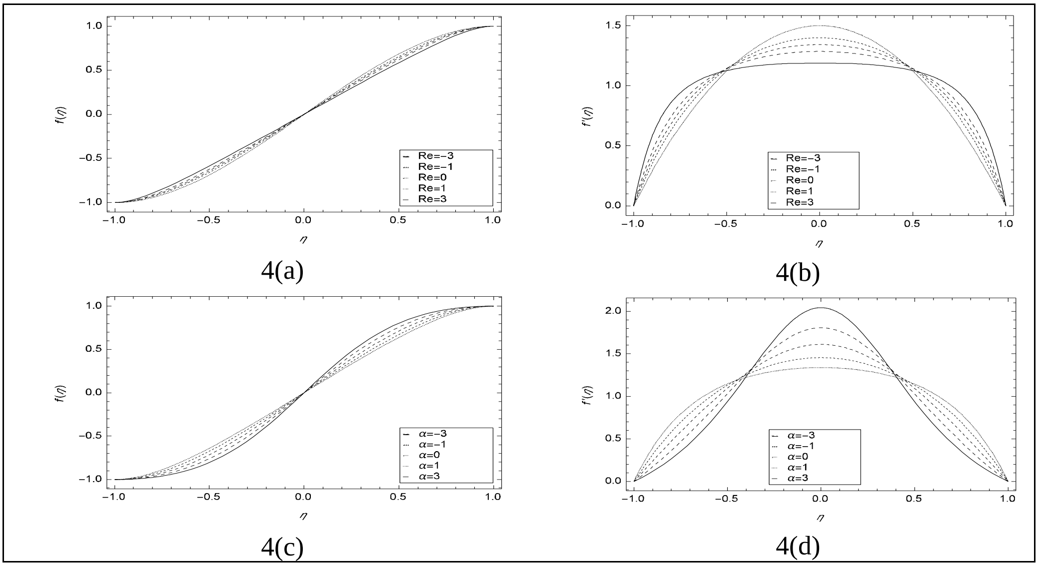

Fig. 4: Effect of the Reynold parameter Re and wall expansion α on velocity profile.

hand, the temperature profiles are remarkably lowered near the disks, and raised for a small region approximately *(-0.4 < η < -0.1)*, On the other hand the concentration profiles are raised only in the lower half of the plane z = 0. For the fixed *Re = -1*, $\phi\ =\ 0.1$ and *Bi=Ec = 0.1*, Fig. 6(c,d) give the influence of the Schmidt number on the concentration profile for $\alpha = \pm 5$. It is noted that for the approaching disks, the concentration profiles are lowered only in the lower half of the plane z = 0 and the trend is reversed for the receding disks.

The effect of the Eckert number on the thermal characteristics of the problem is presented in the Fig. 5(c), for the fixed *Re = -1*, $\phi\ =\ 0.1$ and *Bi=0.1*. For the approaching disks, the temperature profiles are raised across the whole domain whereas, in the other case, the profiles are not much

affected near the disks. As a result the heat transfer rate at the disks is quite low for different Ec in case of $\alpha > 0$.

Finally, the effect of the Biot number on the thermal characteristics of the problem is presents in the Fig.5(d), for fixed *Re = -1*, $\phi\ =\ 0.1$ and *Ec = 0.1*. It shows good result when increase the Biot number temperature decrease, which is beneficial in the field of electronics because we can reduce the heat effects which are harmful for electronic appliance.

5. CONCLUSION

In this paper, we have find the numerically results that how the governing parameters, namely the Reynolds number Re, the nanoparticle volume fraction parameter ϕ, the wall expansion ratio α, the Eckert number Ec, Biot number Bi and the Schmidt number Sc influence the flow and heat transfer characteristics of the unsteady, laminar, incompressible flow of a nanofluid (containing TiO_2 solid nanoparticles) between two orthogonally moving porous coaxial disks with suction. It is noted that the temperature distributions are raised across the whole domain for the approaching disks whereas, in the other case, the distributions are more affected only in the middle of the

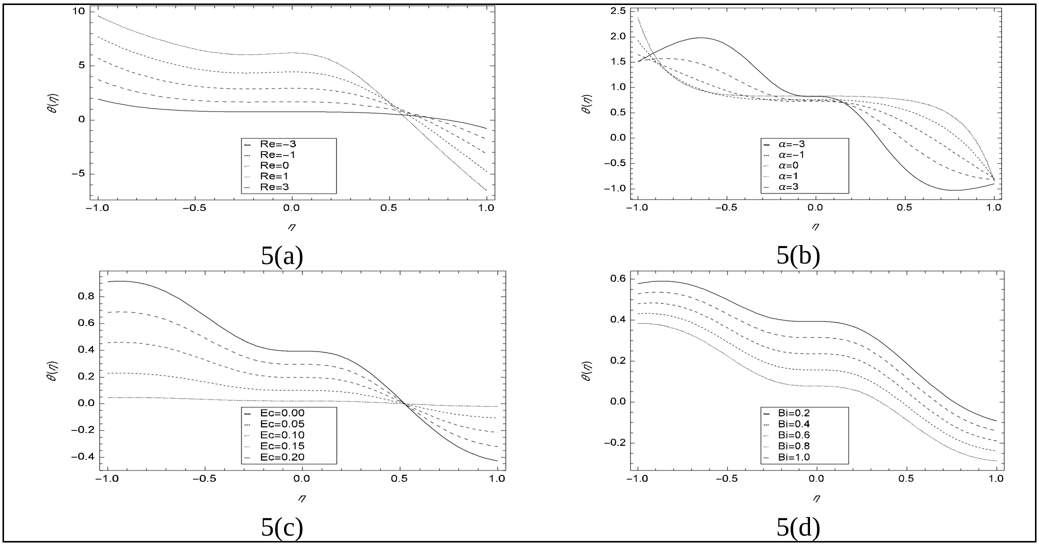

Fig. 5: Effect of the Reynold parameter Re, wall expansion α , Pr and Biot number Bi on temperature profile.

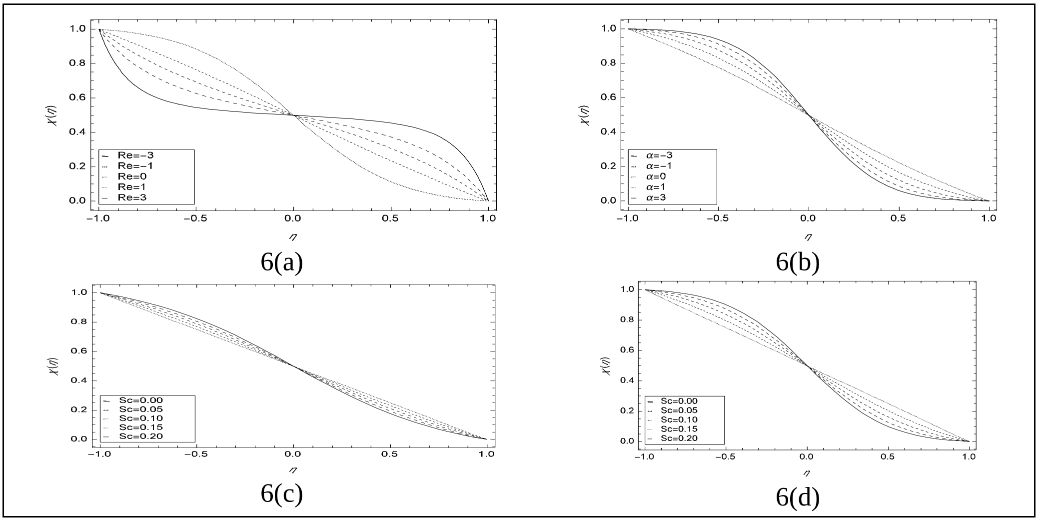

Fig. 6: Effect of the Reynold parameter Re, wall expansion α and Biot Schemidt number on mass transfer profile.

domain, and less influenced near the disks. Similarly, the radial velocity is also noted to be more affected near the disks for the approaching disks. As a result the shear stress and the heat transfer rate acquire higher values for the approaching disks, compared with the receding disks case. Moreover, for both the cases of the disks, the addition of the nanoparticles to the water has no significant influence on the shear stresses and the mass transfer rates at the disks. It is only the heat transfer rate which is affected the most. Finally, it is important to mention here, that the heat transfer characteristics of the nanofluids are not yet fully explored, and that is why complementary works are necessary to identify new and unique applications of these fluids.

Acknowledgments
We are very thankful the journal that they are help us for our research.

References

1. J.C Maxwell, "Electricity and Magnetism", *Oxford UK, Clarendon (1873)*

2. S.U.S Choi, "Enhancing thermal conductivity of fluids with nanoparticles", *American Society of Mechanical Engineers, Fluids Engineering Division (Publication) FED, 231:99--105, (1995).*

3. D. A. Nield and A. Bejan, "Convection in Porous Media", *4th ed. (Springer, New-York), pp. 1–29 (2006)*.

4. Yin, X. Zhang, Flow and heat transfer of nanofluids over a rotating disk with uniform stretching rate in the radial direction, *Propulsion and power research volume 6, 25-30 issue march (2017).*

5. Navid, Ghaziani, Fatemeh H, Convective heat transfer of Al_2O_3 nanofluids in porous media, *Journal of heat transfer volume 139 issue (2017).*

6. T. Hayyat, T. Muhammad, S. A. Shahzad, A. Alsaeedi, On MHD flow of nanofluids due to a rotating disk with slip effect: A numerical study, *Methods Appl. Mech. Energy. 315, 467-477, (2017).*

7. S. P Anjalidevi, T. Ellakipriya, Numerical investigation of slip effects on hydromegnatic flow due to a rotating porous disk in a nanofluids with internal heat absorption, *Journal of Mech. January (2017).*

8. R. A. Mahdi, H.A Mohammed, K. M. Munisamy, and N. H. Saeid, “Review of convection heat transfer and fluid flow in porous media with nanofluid”, *Renewable and Sustainable Energy Reviews 41, 715–734 (2015).*

9. M.Sheikholeslami, M. Hatami, and D.D. Ganji "Analytical investigation of MHD nanofluid flow in a semi-porous channel", *Powder Technology, pages-10 (2013)*.

10. M. Fakour, A. Vahabzadeh, D.D. Ganji, and M. Hatami, "Analytical study of micropolar fluid flow and heat transfer in a channel with permeable walls", *J. of Mol. Liquids pages-7 (2015)*.

11. M. Jashim, O. Uddin, A. Beg, and A. I. MD, "Ismail, Mathematical modeling of radiative hydromagnetic thermosolute nanofluid convection slip flow in saturated porous media", *Hindawi Pub. Corp. pages-11 (2014)*.

12. T. Hayat, M. U Qureshi, and Q. Hussain, "Effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space", *Appl. Math. Modell. 33(4), 1862–1873 (2009)*.

13. N.C. Sacheti, "Application of Brinkman model in viscous incompressible flow thorough a porous channel", *J. of Nume. Method for Heat and Fluid Flow 13(7), 830–848 (2003)*.

14. J. Majdalani, C. Zhou, and C. A. Dawson, "Two dimensional viscous flows between slowly expanding or contracting walls with weak permeability",*Journal of Biomechanics 35, 1399-1403, (2002)*.

Common Region between Associative Algebras and Non-Associative Lie Algebras

Mehsin Jabel Atteya
Al-Mustansiriyah University, College of Education,
Department of Mathematics, Baghdad, IRAQ.
E-mail: mehsinatteya88@gmail.com

AMS 2010 Mathematical Subject Classification : Primary 17B30, Secondary 17 B05.

***Keywords*:** Associative algebra, non-associative Lie algebra, semiprime ring, semisimple Lie algebra, automorphism mapping, commutative area.

DOI : http://dx.doi.org/10.22039/cjcme.2017.06

Abstract: The main purpose of this note is to introduce associative algebra ring and non-associative Lie algebra have a common region. Of this common region we present the area which is commutative.

INTRODUCTION

It has been known, long ago, that some non-associative algebra, for instance Lie algebras, have important applications in Physics. In fact, many important classes of non-associative algebras, as Jordan algebras, have been originated in a Physics frame or have had a big development due to their applications in Physics. This is the case of Kac-Moody algebras (mainly affine algebras), vertex algebras or mutation algebras.

Some other non-associative algebra has been considered in relation to Differential Geometry (see [3]) or differential equations. For instance Lotka-Volterra algebras are associated to quadratic differential equations systems that appear in gas cinetic or populations dynamic (see [1] or [2]). Genetic algebras appear in a Biological context, when one tries to formulate in an algebraic way the transmission of some characters in a random mate of populations (see [4]).One of the most spectacular applications has been achieved with the use of non-associative algebras technics to solve problems in Group theory. The most significant example is the solution to the Restricted Burnside problem using ideas and results of Lie and Jordan algebras. According to [5], there are two important classes of non-associative structures: Lie structures (introduced in 1870 by the Norwegian mathematician Sophus Lie in his study of the groups of transformations) and Jordan structures (introduced in 1932-1933 by the quantum mechanics German specialist Pasqual Jordan in his algebraic formulation of quantum mechanics). Jordan structures also appear in quantum group theory, and exceptional Jordan algebras play an important role in recent fundamental physical theories, namely, in the theory of super-strings (see [6]).

Non-associative algebras are currently a research direction in fashion. In R. Iordanescu [5] new developments show that associative algebras and Lie algebras can be unified at the level of Yang-Baxter structures.

Recently, Brian C. Hall[7] mentions it can be shown that every Lie algebra L can be embedded into some associative algebra R in such way that the bracket on L corresponds to the operation $xy-yx$ in R.

Throughout the present paper R will denote an associative ring with center $Z(R)$. For any $x, y \in R$ the symbol $[x,y]$ represents commentator $xy - yx$. Recall that a ring R is prime if $xRy = 0$ implies $x = 0$ or $y = 0$, and semiprime if $xRx = 0$ implies $x = 0$. In fact, a prime ring is semiprime but the converse is not true in general . A finite-dimensional Lie algebra is a finite-dimensional vector space L over a field F together with a map $[\cdot, \cdot] : L \times L \to L$, with the following properties:

(1) [•, •] is bilinear: [x + v, y] = [x, y] + [v, y] for all x, v, y ∈ L and [αx, y] = α[x, y] for all x, y ∈ L and α ∈ F, [x, y + w] = [x, y] + [x,w] for all x, y, w ∈ L and [x,βy] = β[x, y] for all x, y ∈ L and β ∈ F.

(2) [•, •] is skew-symmetric: [x, x] = 0 for all x ∈ L.

(3) The Jacobi identity holds: [x, [y, z]] + [y, [z, x]] + [z,[x, y]] = 0 for all x, y, z ∈ L.

A Lie algebra L is called semisimple if the only commutative ideal of L is {0}.

Given a Lie algebra L, it be the following descending sequences of ideals of L;

(Central series) $L \supset [L, L] = L^1 \supset [[L, L],L] = L^3 \supset [L^3, L] = L^4 \ldots [L^{k-1}, L] = L^k$ …. (i)

(Derived series) $L \supset [L, L] = L' \supset [L', L'] = L'' \ldots \supset [L^{(k-1)}, L^{(k-1)}] = L^{(k)} \supset \ldots$ (ii).

A Lie algebra is called nilpotent (resp. solvable) if $L^k = 0$ (resp. $L^{(k)} = 0$) for k sufficiently large. Every nilpotent Lie algebra is soluble, a Lie algebra L over a field F is called semisimple if it has no soluble ideals other than 0. A Lie algebra L is associative if [L, [L, L]] = 0.

In fact, any abelian or nilpotent Lie algebra is solvable Lie algebra and every solvable Lie algebra is semisimple Lie algebra. An R-module M is called Artinian if it satisfies the descending chain condition (DCC) for submodules. A ring R is called left-Artinian if RR is an Artinian module. A ring R with 1 is semisimple, or left semisimple to be precise, if the free left R-module underlying R is a sum of simple R-module. In this note we will determine whether the associative algebra ring and non-associative Lie algebra have a common region.

In other words, we found the area which is commutative where the working becomes easier by transports some problems from non-associative Lie algebra to that area. In fact, this area is the center of semiprime rings.

2. THE MAIN RESULTS

Theorem 2.1

Let R be associative algebra semiprime ring with Z(R), L is non associative semisimple Lie algebra and λ be automorphism mapping such that λ : L→ Z(R), then Z(R) is common region between L and R.

Proof: Let x ∈ L, since the additive mapping λ : L→ Z(R), that means λ(x) ∈ Z(R).

Where we knowledge that Z(R) is ideal of R. Then x ∈ Z(R), where λ be automorphism mapping , that is meaning

[x, r] = 0 for all r ∈ R.

Replacing r by λ(y), y ∈ L.

We achieve the following relation

[x, λ(y)] = 0 for all x, y ∈ L.

According to the fact that R has no non-zero nilpotent left (right/two-sided) ideals, with the note that x, y ∈ Z(R) ⊂ R and λ is automorphism mapping , we obtain

$L^1 = [L, L] = 0.$

$L^2 = [L, L^1] = [L,[L, L]] = 0.$

$L^3 = [L, L^2] =[L, [L, [L, L]]] = 0, \ldots., L^n = 0$, for some $n \geq 1$.

Then L is a nilpotent Lie algebra, where any abelian or nilpotent Lie algebra is solvable Lie algebra, which means L is non associative semisimple Lie algebra as required.

It is clear we have non associative semisimple Lie algebra inside Z(R) which considers associative algebra.

In 1974, Thomas W. Hungerford [9] mention of his book in page (446), left Artinian semiprime rings are also semisimple rings.

Note that in paper[8], the author proved his Corollary (17.5), which state for any ring R the following are equivalent:

(i) R is semiprime.

(ii) R has no non-zero nilpotent left (right/two-sided) ideals.

In other words, according to above result, the left- Artinian semisimple rings has no non-zero nilpotent left (right/two-sided) ideals.

Therefore by use same technique in Theorem 2.1, we can prove the following results.

Theorem 2.2

Let R be associative algebra left Artinian semisimple ring with Z(R) is an ideal of R and L is non associative semisimple Lie algebra and λ be automorphism mapping such that $\lambda : L \rightarrow Z(R)$, then L and R have common region.

Corollary 2.3

Let R be associative algebra prime ring with Z(R), L is non associative semisimple Lie algebra and λ be automorphism mapping such that $\lambda : L \rightarrow Z(R)$, then Z(R) is common region between L and R.

Acknowledgments

The author is greatly indebted to the referee for his careful reading the paper.

References

1. N. C. Hopkins, Quadratic differential equations in graded algebras. In Nonassociative Algebra and its Applications (S. Gonzálezed.), Math. Appl. 303, Kluwer Acad. Publ.,(1994), pp. 179–182.

2. L. Markus, Quadratic differential equations and non-associative algebras, Annals of Math. Stud., 45, (1960), pp. 185–213.

3. A. Elduque and H.C. Myung, The reductive pair (B4, B3) and affine connections on S15. J. Algebra 227, no. 2, (2000), pp.504–531, 2000.

4. M. L. Reed, Algebraic structures of genetic inheritance. Bull. Amer. Math. Soc.34, (1997), pp.107–130.

5. R. Iordanescu, The associativity in present mathematics and present physics, presentation – Bucharest, 2014.

6. R. Iordanescu, 2011, arXiv preprint math-DG/1106.4415.

7. C. Brian Hall, Lie groups, Lie algebras, and Representations, An Elementary Introduction, Graduate Texts in Mathematics, Springer-Verlag New York, Inc., 2003.

8. Frank W. Anderson, Lectures on non-commutative rings, Mathematics 681 University of Oregon, Oregon, 2002.

Rahpoyan Andisheh & Honar cultural and artistic Institute

Activities :

- Film making
- Implementation of cultural & artistic projects
- Cultural and artistic trade
- Organizing to Perform concerts ,theaters , exhibitions
- Leading artistic lessons (Painting , Graphic designing ,Photography ,…)
- Inviting artistic groups for performance

Director : Dr. S.M.Ali Raeisian

Email : rpandishehonar@gmail.com

https://t.me/rpandishehonar

نحوه پذیرش مقالات

نشریه تخصصی CJCME دستاورد و نتایج تحقیقات اساتید و پژوهشگران رشته های فنی مهندسی و علوم پایه را منتشر می کند .

از محققانی که برای این نشریه مقاله تهیه می کنند درخواست می شود ضمن رعایت دقیق مفاد آئین نامه نگارش نشریه تخصصی CJCME ، مقالات خود را در یک نسخه فایل Word (حداکثر 20 صفحه) از طریق پست الکترونیک CJCMEmail@gmail.com ارسال دارند .

تمامی مقالات توسط داوران ذیصلاح ارزشیابی می شوند و نشریه تخصصی CJCME در پذیرش ، عدم پذیرش ، حذف و یا کوتاه نمودن مقالات برای چاپ آزاد است .

فقط مقالاتی جهت انتشار در نشریه تخصصی CJCME مورد بررسی قرار می گیرند که قبلاً در نشریات علمی پژوهشی دیگر به چاپ نرسیده باشند .

نویسندگان مقالات مسئول نوشته ها و نظرات خود هستند و آراء و نظریات آنان الزاماً نظر اعضای هیات تحریریه نیست .

Editorial Board

Dr. Seyed Mohammadali Raeisian , Iran

Fields of Interests: *Differential Equations , Elliptic Partial Differential Equations , Numerical Methods*

Professor Armenak Babayan , National Polytechnic University of Armenia , Armenia

Fields of Interests: *Differential Equations , Partial Differential Equations , Elliptic Partial Differential Equations , Integral Equations , Singular Integral Equations , Wiener-Hopf Equations , Numerical Methods , Complex Analysis*

Professor Esmaiel Babolian , Kharazmi University , Iran

Fields of Interests : *Numerical Analysis , Numerical solution of Partial Differential Equations , Numerical solution of Integral Equations , Approximation Theory , Galerkin Method for Integral and Integro-Differential Equations*

Dr. Elmar Diederichs , Moscow Institute of Physics and Technology , Russia

Fields of Interests : *Numerical Analysis , Machine Learning , Optimization and Non-parametric Statistics*

Professor Levon Gevorgyan , National Polytechnic University of Armenia , Armenia

Fields of Interests : *Functional analysis , Operator Theory , Functional Analysis , Harmonic Analysis , Spectral Theory , Krylov Subspace Methods , Paranormal Operators , Numerical Range*

Professor Henk Koppelaar , **Fac. Electric Engineering, Mathematics and Computer Science, Delft University of Technology Delft, The Netherlands**

Fields of Interests : *Neural Networks , Data Mining , AI , Automated Programming, Automated Discovery ,Computer Algebra, Discrete Mathematics, Experimental Mathematics, Fractional Calculus, Mathematical Physics, Numerical Analysis, Proof Automation .*

Professor Vanya Alexandrovich Mirzoyan , **Applied Mathematics and Informatics Faculty, National Polytechnic University of Armenia , Armenia**

Fields of Interests : *Geometry and Topology , Differential Geometry , Riemannian Geometry , Geometry of Riemannian Submanifolds*

Dr. Antti Rasila , **Aalto University, Helsinki, Finland**

Fields of Interests : *Complex Analysis, Partial Differential Equations, Computational Methods , Geometric function theory , Elliptic PDEs*

Professor Masoud Sabbaghan , **The College of Sciences of the University of Tehran , Iran**

Fields of Interests : *Fractal , Dynamical Systems , Fixed Point Theorem*

Professor Hashem Saberi Najafi , **The University of Guilan & Islamic Azad University of Lahijan branch , Iran**

Fields of Interests : *Numerical Analysis , Numerical Linear Algebra , Numerical Ordinary and Partial Differential Equations , Numerical Modeling of the Oceans*

Caspian Journal of Computational & Mathematical Engineering (CJCME) : اسم نشریه
دوره انتشار : دوفصلنامه
زبان نشریه : انگلیسی ، فارسی ، روسی ، ارمنی.
شمارگان : 500 نسخه
صاحب امتیاز : دکتر سید محمد علی رئیسیان
مدیر مسئول : دکتر سید محمد علی رئیسیان
رئیس هیت تحریریه : دکتر سید محمد علی رئیسیان
شماره مجوز نشریه : 75729

محل انتشار : رودسر
نشانی : گیلان ، رودسر ، صندوق پستی 1165 -44815 دکتر سید محمد علی رئیسیان
پست الکترونیک : smaraissian53@gmail.com , CJCMEmail@gmail.com
تماس : (+98)911 143 4988 , (+98) 13 - 42613646
وب سایت : https://caspianjcme.com

ویراستاری و صفحه بندی و تنظیم : موسسه فرهنگی و هنری ره پویان اندیشه و هنر
rpandishehonar@gmail.com

قیمت : 20000 تومان

تاریخ انتشار : شهریور 1396

کلیه حقوق اعم از چاپ و تکثیر به هر شکل و میزان، نسخه برداری ، ترجمه و جز اینها برای نشریه تخصصی
Caspian Journal of Computational & Mathematical Engineering (CJCME)
محفوظ است. متخلفین بر اساس قانون حمایت از مولفین و مصنفین و هنرمندان تحت تعقیف قرار خواهند گرفت.

Copyright © 2017 CJCME
All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the written permission of the copyright owner.


ISSN : 2476-4418
CASPIAN
Computational & Mathematical Engineering
Caspian Journal
of
Computational & Mathematical Engineering
2016
No.1
Caspian Journal of Computational & Mathematical Engineering (CJCME) • 2016 • No . 1
CASPIAN
Computational & Mathematical Engineering
Caspian Journal of Computational & Mathematical Engineering (CJCME)
2016 • No . 1
Contents
Defect Numbers of the Dirichlet Problem for Higher Order Partial Differential Equations in the Unit Disc , A. H. Babayan, V.A. Babayan ……4
A unified approach to the data fitting problem , L.Gevorgyan ……20
Introduction to phantom graph , Philip LLOYD ……31
A geometrical Isomorphism and the relativity of geometry , George Mpantes ……42
Convex geometric reasoning for crystalline energies , Thaicia Stona ……51


Caspian Journal
of
Computational & Mathematical Engineering
2016
No.2
Contents