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In the name of God

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Numerical Solution of Fractional Neutral Functional Differential Equations by A shifted Chebyshev Computational Matrix

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ABSTRACT. In this article, we develop a direct solution technique for solving Fractional Neutral Functional-Differential Equations (FNFDEs) using a matrix method based upon the shifted Chebyshev tau and shifted Chebyshev collocation

method. The fractional derivatives are described in the Caputo sense. The main characteristic behind the approach using this technique is that it reduces the problems to a system of algebraic equations. The results reveal that the proposed method is very effective and simple.

1. INTRODUCTION

The origin of fractional calculus goes back to Leibniz and Newton in the seventieth century. Mathematical folklore sets the birth of the concept of fractional calculus in the year 1695 by the answer to a question raised by L'Hôpital (1661-1704) to Leibniz (1646-1716), in which he sought the meaning of Leibniz's notation $\frac{d^n y}{dx^n}$ for derivatives if $n = \frac{1}{2}, \frac{1}{3}, \ldots$. In his reply, dated 30 September 1695, Leibniz wrote to L'Hôpital (quoting from [1]) "this is an apparent paradox from which, one day, useful consequence will be drawn…".

The first book devoted exclusively to the subject of fractional calculus, is the book by Oldham and Spanier [1] published in 1974. A much later book, by Podlubny [2], is from 1996 and the book by Kilbas, Srivastava and Trujillo [3] appeared in 2006.

In recent years, it has turned out that many phenomena in viscoelasticity, fluid mechanics, biology, chemistry, acoustics, control theory, psychology and other areas of science can be successfully modeled by the use of fractional order derivatives [1, 2, 3].

Chebyshev polynomials are interesting examples of orthogonal polynomials and use for approximating other functions. They are widely used in many areas of numerical analysis, in the context of numerical methods for solving ordinary and partial differential equations, Chebyshev polynomials are so-called spectral or pseudospectral method.

Spectral methods have become increasingly popular in recent years, especially since the development of fast transform methods, with applications in numerical weather prediction, numerical simulations of turbulent flows, and other problems where high accuracy is desired for complicated solutions. Spectral methods involve representing the solution to a problem as a truncated sense of known functions of the independent variables [4].

There are three most commonly used spectral versions, namely the Galerkin-type, tau and collocation methods. Among them, the spectral collocation or pseudospectral method is particularly attractive due to its economy.

Our main aim of this study is to propose the Chebyshev operational matrix and Chebyshev's collocation approximation to solve a new class of functional-differential equation with fractional orders on the interval $(0, L)$ to obtain the numerical solution. We organized this paper as follows: In section 2, some necessary definitions and some relevant properties of Chebyshev polynomials are introduced. In chapter 3 the operational matrix of derivatives is developed with its accompanying Chebyshev type polynomials in Ch.4 of course we apply the newly developed method subsequently in Ch.6.

This application reveals the very high accuracy of the newly developed method.

2. BACKGROUND AND DEFINITIONS

In this section, we state definitions of fractional calculus, as needed in the sequel.

The Caputo fractional derivatives of order v are defined as

$$D^v u(x) = J^{m-v} D^m u(x) = \frac{1}{\Gamma(m-v)} \int_0^x (x-s)^{m-v-1} \frac{d^m}{ds^m} u(s) ds, \qquad (2.1)$$

Where D^m is the classical differential operator of order m and m is the smallest integer greater than v. For the Caputo derivative we have [5,6]

$$D^v x^\mu = \begin{cases} 0 & for\ \mu \in N_0,\ \mu < \lceil v \rceil \\ \frac{\Gamma(\mu+1)}{\Gamma(\mu+1-v)} x^{\mu-v}, & \mu \in N_0, \mu \geq \lceil v \rceil\ or\ \mu \notin N, \mu > \lfloor v \rfloor \end{cases} \qquad (2.2)$$

Where $\lceil v \rceil$ is the ceiling function and $\lfloor v \rfloor$ is the floor function.

The Caputo fractional differentiation is a linear operation

$$D^{v}\big(\lambda f(x)+\eta g(x)\big)=\lambda D^{v}f(x)+\eta D^{v}g(x), \qquad (2.3)$$

Where λ and η are constants.

2.2 properties of shifted Chebyshev polynomials

The well known Chebyshev polynomials $T_n(x)$, defined on the interval $I=[-1,1]$ have the following properties

$$T_0(x)=1, T_1(x)=x, \dots, T_{n+1}(x)=2xT_n(x)-T_{n-1}(x), \qquad (2.4)$$

The weight function for these polynomials is $\omega(x)=\frac{1}{\sqrt{1-x^2}}$ and the weighted space is equipped with the following inner product

$$(u,v)=\int_{-1}^{1}u(x)v(x)\omega(x)dx,$$

In order to use these polynomials on the interval $x\in[0,L]$ we define the so-called shifted Chebyshev polynomials by introducing the change of variable $t=\frac{2x}{L}-1$. Let $T_n^*(x)$ denote the shifted Chebyshev polynomials.

Then $T_n^*(x)$ can be obtained as follows

$$T_{n+1}^*(x)=2\left(\frac{2x}{L}-1\right)T_n^*(x)-T_{n-1}^*(x),\ \ i=1,2,\dots \qquad (2.5)$$

Where $T_0^*=1$ and $T_1^*(x)=\frac{2x}{L}-1$.

The analytic form of the shifted Chebyshev polynomials $T_i^*(x)$ of degree i is given by

$$T_i^*(x) = i\sum_{k=0}^{i}(-1)^{i-k}\frac{(i+k-1)!2^{2k}}{(i-k)!(2k)!L^k}x^k \tag{2.6}$$

Note that $T_i^*(0) = (-1)^i$ and $T_i^*(L) = 1$. The orthogonality condition is

$$\int_0^L T_j^*(x)T_k^*(x)\omega_L(x)dx = h_j\delta_{jk}, \tag{2.7}$$

where $\omega_L(x) = \frac{1}{\sqrt{Lx-x^2}}$ and $h_j = \frac{b_j}{2}\pi$, $b_0 = 2, b_j = 1(j \geq 1)$.

A function $u(x)$, square integrable in $(0, L)$, can be expressed in terms of shifted Chebyshev polynomials as

$$\boldsymbol{u}(x) = \sum_{j=0}^{\infty} c_j T_j^*(x),$$

where the coefficients c_j are given by

$$c_j = \frac{1}{h_j}\int_0^L u(x)T_j^*(x)\omega_L(x)dx,\ \ j = 0,1,2,\dots \tag{2.8}$$

In practice, only the first (m+1)-terms shifted Chebyshev polynomials are considered. Hence we have

$$u(x) = \sum_{j=0}^{m} c_j T_j^*(x) = C^T\psi(x), \tag{2.9}$$

where C is a shifted Chebyshev coefficient vector and ψ is a shifted Chebyshev vector as the following

$$C^T = [c_0, c_1, \dots, c_m], \tag{2.10}$$

$$\psi(x) = [T_0^*(x), T_1^*(x), \dots, T_m^*(x)]^T.$$

3. CHEBYSHEV OPERATIONAL MATRIX OF THE FRACTIONAL DERIVATIVE

Theorem1

The derivative of the vector $\psi(x)$ can be expressed by

$$\frac{d\psi(x)}{dx} = \Lambda^{(1)}\psi(x), \tag{3.1}$$

where $\Lambda^{(1)}$ is the $(m+1)\times(m+1)$ operational matrix of derivative given by

$$\Lambda^{(1)} = (d_{ij}) = \begin{cases} \frac{4i}{L} & for\ j = i-k > 0 \begin{cases} k = 1,3,\dots,m & if\ m\ odd. \\ k = 1,3,\dots,m-1 & if\ m\ even. \end{cases} \\ \frac{2i+3j}{L} & for\ j = i-k = 0 \\ 0 & otherwise, \end{cases}$$

where i, j are numbers of the rows and columns respectively.

For example, m=10 on the interval [0,1] we have

$$\Lambda^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 20 & 0 & 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 24 & 0 & 24 & 0 & 24 & 0 & 0 & 0 & 0 \\ 0 & 0 & 28 & 0 & 28 & 0 & 28 & 0 & 0 & 0 \\ 0 & 32 & 0 & 32 & 0 & 32 & 0 & 32 & 0 & 0 \\ 0 & 0 & 36 & 0 & 36 & 0 & 36 & 0 & 36 & 0 \end{bmatrix}$$

By using equation (11) for $n \in N$, we have

$$\frac{d^n\psi(x)}{dx^n} = (\Lambda^{(1)})^n\psi(x). \tag{3.2}$$

Theorem 2

Let $\psi(x)$ be a shifted Chebyshev vector defined in (10) and also $v > 0$ then

$$D^v\psi(x) \simeq \Lambda^{(v)}\psi(x), \tag{3.3}$$

where $\Lambda^{(v)}$ is the $(m+1)\times(m+1)$ computational matrix of fractional derivative of order v in the Caputo sense and is defined as follows

$$\Lambda^{(v)} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \\ \sum_{k=\lceil v\rceil}^{\lceil v\rceil} \Omega_{\lceil v\rceil,0,k} & \sum_{k=\lceil v\rceil}^{\lceil v\rceil} \Omega_{\lceil v\rceil,0,k} & \cdots & \sum_{k=\lceil v\rceil}^{\lceil v\rceil} \Omega_{\lceil v\rceil,0,k} \\ \vdots & \vdots & \cdots & \vdots \\ \sum_{k=\lceil v\rceil}^{\lceil v\rceil} \Omega_{\lceil v\rceil,0,k} & \sum_{k=\lceil v\rceil}^{\lceil v\rceil} \Omega_{\lceil v\rceil,0,k} & \cdots & \sum_{k=\lceil v\rceil}^{\lceil v\rceil} \Omega_{\lceil v\rceil,0,k} \\ \vdots & \vdots & \cdots & \vdots \\ \sum_{k=\lceil v\rceil}^{\lceil v\rceil} \Omega_{\lceil v\rceil,0,k} & \sum_{k=\lceil v\rceil}^{\lceil v\rceil} \Omega_{\lceil v\rceil,0,k} & \cdots & \sum_{k=\lceil v\rceil}^{\lceil v\rceil} \Omega_{\lceil v\rceil,0,k} \end{bmatrix}$$

Where $\Omega_{i,k,j}$ is given by

$$\Omega_{i,j,k} = \begin{cases} \frac{i}{\sqrt{\pi}}(-1)^{i-k}\frac{(i+k-1)!2^{2k}k!\Gamma(k-v+\frac{1}{2})}{(i-k)!(2k)!L^v(\Gamma(k-v+1))^2}, & j = 0 \\ \frac{ij}{\sqrt{\pi}}\sum_{l=0}^{j}(-1)^{i+j-k-l}\frac{(i+k-1)!2^{2(k+l)+1}k!(j+l-1)!\Gamma(l+k-v+\frac{1}{2})}{(i-k)!(2k)!L^v(k-v)!(j-l)!(2l)!(k+l-v)!}, & j > 0 \end{cases} \tag{3.4}$$

After some lengthy manipulation, $\Omega_{i,j,k}$ can be put in the following for

$$\Omega_{i,j,k} = \frac{(-1)^{i-k}2i\,(i+k-1)!\Gamma(k-v+\frac{1}{2})}{b_j\,\Gamma\left(k+\frac{1}{2}\right)(i-k)!\Gamma(k-v-j+1)\Gamma(k+j-v+1)L^v}, \tag{3.5}$$

Note that in $\Lambda^{(v)}$, the first $\lceil v \rceil$ rows are all zero, see[5].

Proof:

$$D^v T_i^*(x) = i \sum_{k=0}^{i} (-1)^{i-k} \frac{(i+k-1)!2^{2k}}{(i-k)!(2k)!L^k} D^v x^k = i \sum_{k=\lceil v \rceil}^{i} (-1)^{i-k} \frac{(i+k-1)!2^{2k}\Gamma(k+1)}{(i-k)!(2k)!L^k\Gamma(k+1-v)} x^{k-v}, \quad (3.6)$$

Now, we approximate x^{k-v} by $(m+1)$ terms of shifted Chebyshev series, we have

$$x^{k-v} \simeq \sum_{j=0}^{k-v} c_j T_j^*(x), \quad (3.7)$$

Where

$$c_j = \frac{1}{h_j} \int_0^L x^{k-v} j \sum_{l=0}^{j} (-1)^{j-l} \frac{(j+l-1)!2^{2l}}{(j-l)!(2l)!L^k} x^l \frac{1}{\sqrt{Lx-x^2}} dx = \frac{j}{h_j} \sum_{l=0}^{j} (-1)^{j-l} \frac{(j+l-1)!2^{2l}}{(j-l)!(2l)!L^k} \int_0^L x^{k-v} x^l \frac{1}{\sqrt{Lx-x^2}} dx = \frac{j}{h_j} \sum_{l=0}^{j} (-1)^{j-l} \frac{(j+l-1)!2^{2l}}{(j-l)!(2l)!} \times L^{l-v} \times \frac{\Gamma(l+k-v+\frac{1}{2})}{(l+k-v)!} \times \sqrt{\pi}, \quad (3.8)$$

After employing equations (3.6)-(3.8) we have

$$D^v T_i^*(x) = \frac{ij\sqrt{\pi}}{h_j} \sum_{k=\lceil v \rceil}^{i} \sum_{j=0}^{k-v} \sum_{l=0}^{j} (-1)^{i+j-k-l} \frac{(i+k-1)!2^{2(k+l)}k!(j+l-1)!\Gamma\left(l+k-v+\frac{1}{2}\right)}{(i-k)!(2k)!L^v(k-v)!(j-l)!(2l)!(k+l-v)!} T_j^*(x).$$

4. NORMALIZED SHIFTED CHEBYSHEV POLYNOMIALS FOR SOLVING FNFDE

Consider the following Fractional Neutral Functional Differential Equation from [6]

$$D^{v}\big(u(x)+a(x)u(p_m x)\big)=\beta u(x)+\sum_{n=0}^{m-1} b_n(x)D^{\gamma_n}u(p_n x)+f(x),\ \ x\geq 0 \quad (4.1)$$

with the initial conditions

$$\sum_{r=0}^{n-1} c_{ir}u^{(r)}(0)=\lambda_i\ . \quad (4.2)$$

where $a, b_n(n=0,1,2,\dots,m-1)$ are given analytical functions, $m-1<v\leq m$, $0<\gamma_0<\gamma_1<\cdots<\gamma_{m-1}<v$ and $\beta, p_n, c_{in}, \lambda_i$ denote given constants with $0<p_n<1\ (n=0,1,\dots,m)$.

To solve equation (4.1) we apply equation (2.3) on equation (4.1) and rewrite as

$$D^{v}u(x)+a(x)D^{v}u(p_m x)=\beta u(x)+\sum_{n=0}^{m-1} b_n(x)D^{\gamma_n}u(p_n x)+f(x), \qquad x\geq 0 \quad (4.3)$$

Then we approximate $u(x), f(x),\ D^{v}u(x), D^{v}u(p_m x), D^{\gamma_n}u(p_n x)$, by the shifted Chebyshev polynomials as

$$u(x)\simeq \sum_{i=0}^{m} c_i T_i^*(x)=C^T\psi(x). \quad (4.4)$$

$$f(x)\simeq \sum_{i=0}^{m} f_i T_i^*(x)=F^T\psi(x). \quad (4.5)$$

$$D^{v}u(x)\simeq C^T D^{v}\psi(x)=C^T\Lambda^{v}\psi(x). \quad (4.6)$$

$$D^{v}u(p_m x)\simeq C^T D^{v}\psi(p_m x)=C^T\Lambda^{v}\psi(p_m x). \quad (4.7)$$

$$D^{\gamma_n}u(p_n x)\simeq C^T D^{v}\psi(p_n x)=C^T\Lambda^{v}\psi(p_n x). \quad (4.8)$$

Where vector $F = [f_0, \dots, f_m]^T$ is known but $C = [c_0, \dots, c_1]^T$is an unknown vector.

By substituting equations (4.4)-(4.8) in equation (4.3) we get

$$C^T \Lambda^v \psi(x) + a(t) C^T \Lambda^v \psi(p_m x) = \beta C^T \psi(x) + \sum_{i=0}^{m} b_n(x) C^T \Lambda^v (p_n x) + f(x). \tag{4.9}$$

Now, we collocate equation (4.9) at (m-n+1) points x_δ, $\delta = 0,1, \dots, m-n$, as

$$C^T \Lambda^v \psi(x_\delta) + a(t) C^T \Lambda^v \psi(p_m x_\delta) = \beta C^T \psi(x_\delta) + \sum_{i=0}^{m} b_n(x) C^T \Lambda^v (p_n x_\delta) + f(x_\delta). \tag{4.10}$$

For suitable collocation points we use roots of shifted Chebyshev polynomials $T^*_{m-n+1}(x)$. Equation (4.10), together with n equation of boundary conditions, give (m+1) linear or nonlinear algebraic equations which can be solved using Gauss elimination method and Newton's iterative method respectively.

5.COMPUTATIONAL OF THE ERROR FUNCTIONS

In this section, an error estimation for the approximate solution of equation (4.1) with supplementary conditions is obtained.

Let us rewrite equation (2.9) as $u_n(x) = \sum_{j=0}^{m} c_j T_j^*(x)$ then its truncation error is

$e_n(t) = u(x) - u_n(x) = \sum_{j=0}^{\infty} c_j T_j^*(x) - \sum_{j=0}^{m} c_j T_j^*(x) = \sum_{j=m+1}^{\infty} c_j T_j^*(x),$

Where $u(x)$ is the exact solution of equation (4.1). Therefore, $u_n(x)$ satisfies the following equations

$D^v\big(u_n(x) + a(x)u_n(p_m x)\big) = \beta u_n(x) + \sum_{n=0}^{m-1} b_n(x) D^{\gamma_n} u_n(p_n x) + f(x) + H_n(x),\ \ x \geq 0, \sum_{r=0}^{n-1} c_{ir} u_n^{(r)}(0) = \lambda_i,\ m-1 < v \leq m\ , 0 < \gamma_0 < \gamma_1 < \cdots < \gamma_{m-1} < v.$ (5.1)

The perturbation term $H_n(x)$ can be obtained by substituting the computed solution $u_n(t)$ into the equation

$H_n(x) = D^v\big(u_n(x) + a(x)u_n(p_m x)\big) - \beta u_n(x) - \sum_{n=0}^{m-1} b_n(x) D^{\gamma_n} u_n(p_n x) - f(x).$

By subtracting equation (5.1) from equation (4.1), we have

$$D^v\Big(\big(u(x) - u_n(x)\big) + a(x)\big(u(p_m x) - u_n(p_m x)\big)\Big)$$

$$= \beta\big(u(x) - u_n(x)\big) +$$

$$\sum_{n=0}^{m-1} b_n(x) D^{\gamma_n}\big(u(p_n x) - u_n(p_n x)\big) + \big(f(x) - f_n(x)\big)$$

$$- H_n(x), \sum_{r=0}^{n-1} c_{ir}\left(u^{(r)}(0) - u_n^{(r)}(0)\right) = 0$$

Or

$$D^v\big(e_n(x) + a(x)(e_n(p_m x)\big)$$

$$= \beta(e_n(x)) + \sum_{n=0}^{m-1} b_n(x) D^{\gamma_n}\big(e_n(p_n x)\big) - H_n(x), \sum_{r=0}^{n-1} c_{ir}\left(e_n^{(r)}(0)\right) = 0\ .$$

6. NUMERICAL EXAMPLES

In this section some numerical examples are given to clarify the high accuracy of the method.

Example 6.1.

Consider the following FNFDEs [6]

$$u^{\frac{5}{2}}(x) = u(x) + u^{\frac{1}{2}}\left(\frac{x}{2}\right) + u^{\frac{3}{2}}\left(\frac{x}{3}\right) + \frac{1}{2}u^{\frac{5}{2}}\left(\frac{x}{4}\right) + \frac{\Gamma(5)}{\Gamma(\frac{5}{2})}\left(x^{\frac{3}{2}}\right) - \frac{\Gamma(4)}{\Gamma\left(\frac{3}{2}\right)}x^{\frac{1}{2}} - x^4 + x^3$$
$$- \frac{\Gamma(5)}{\Gamma\left(\frac{9}{2}\right)}(\frac{x}{2})^{\frac{7}{2}} + \frac{\Gamma(4)}{\Gamma\left(\frac{7}{2}\right)}(\frac{x}{2})^{\frac{5}{2}} - \frac{\Gamma(5)}{\Gamma\left(\frac{7}{2}\right)}(\frac{x}{3})^{\frac{5}{2}} + \frac{\Gamma(4)}{\Gamma\left(\frac{5}{2}\right)}(\frac{x}{3})^{\frac{3}{2}}$$
$$- \frac{\Gamma(5)}{2\Gamma\left(\frac{5}{2}\right)}(\frac{x}{4})^{\frac{3}{2}} + \frac{\Gamma(4)}{2\Gamma\left(\frac{3}{2}\right)}(\frac{x}{4})^{\frac{1}{2}}, \quad x \in [0,1]$$

With

$$u(0) = 0, \qquad u'(0) = 0, \quad u''(0) = 0,$$

The exact solution is $u(x) = x^4 - x^3$.

We implement the suggested method with m=4, using Theorem (1), Theorem (2) and equation (4.9) we have

$$\Lambda^{\frac{1}{2}} = \frac{8}{\pi^{\frac{3}{2}}}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & \frac{2}{3} & -\frac{2}{15} & \frac{2}{35} & -\frac{2}{63} \\ -\frac{4}{9} & \frac{8}{5} & \frac{8}{7} & -\frac{8}{27} & \frac{8}{55} \\ \frac{33}{25} & -\frac{2}{21} & \frac{74}{45} & \frac{582}{385} & -\frac{362}{819} \\ -\frac{272}{441} & \frac{416}{225} & \frac{32}{231} & \frac{608}{351} & \frac{1504}{825} \end{bmatrix},$$

$$\Lambda^{\frac{3}{2}} = \frac{64}{\pi^{\frac{3}{2}}}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & \frac{2}{3} & -\frac{2}{15} & \frac{2}{35} & -\frac{2}{65} \\ -\frac{2}{3} & \frac{12}{5} & \frac{12}{7} & -\frac{4}{9} & \frac{12}{55} \\ \frac{116}{25} & \frac{8}{7} & \frac{136}{45} & \frac{1208}{385} & -\frac{776}{819} \end{bmatrix},$$

$$\Lambda^{\frac{5}{2}} = \frac{256}{\pi^{\frac{3}{2}}}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & -\frac{2}{5} & \frac{6}{35} & -\frac{2}{21} \\ -\frac{8}{3} & \frac{48}{5} & \frac{48}{7} & -\frac{16}{9} & \frac{48}{55} \end{bmatrix},$$

$$\Lambda^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 6 & 0 & 12 & 0 & 0 \\ 0 & 16 & 0 & 16 & 0 \end{bmatrix},$$

$$C^T\Lambda^{\frac{5}{2}}\psi(x_\delta) - C^T\psi(x_\delta) - C^T\Lambda^{\frac{1}{2}}\psi\left(\frac{x_\delta}{2}\right) - C^T\Lambda^{\frac{3}{2}}\psi\left(\frac{x_\delta}{3}\right) - \frac{1}{2}C^T\Lambda^{\frac{5}{2}}\psi\left(\frac{x_\delta}{4}\right) = F^T\psi(x_\delta). \tag{6.1}$$

Where $C=[c_0,c_1,c_2,c_3,c_4]^T$, $F=[f_0,f_1]$ and

$\psi(x)=[1,2x-1,8x^2-8x+1,32x^3-48x^2+18x-1,128x^4-256x^3+160x^2-32x+1]^T$.

With $\delta=0{,}1$ where x_δ are roots of the shifted Chebyshev polynomials $T_2^*(x)$ we have $x_0=\frac{8+\sqrt{32}}{16}$, $x_1=\frac{8-\sqrt{32}}{16}$

for these points we obtain $f_0=9.23034294$, $f_1=-0.94463255$ and for collocation points we have

Now, by substituting in equation (6.1) we obtain

$$188.587044c_3-2.183966c_1-7.3155c_2-c_0+439.849290c_4=f_0\ , \tag{6.2}$$

$$0.14294c_1-c_0-1.5373991c_2+79.943438c_0-448.8692846c_4=f_1\ , \tag{6.3}$$

And for the initial conditions

$u(0)=C^T\psi(0)$, $u'(0)=C^T\Lambda^{(1)}\psi(0)$, $u''(0)=C^T\Lambda^{(2)}\psi(0)$,

For the above initial conditions we obtain the following equations

$$c_0-c_1+c_2-c_3+c_4=0\ , \tag{6.4}$$

$$2c_1-8c_2+18c_3-32c_4=0, \tag{6.5}$$

$$16c_2-96c_3+320c_4=0, \tag{6.6}$$

Finally, after solving the system (6.2)-(6.6), we get

$c_0=-0.02793055$, $c_1=-0.01481252$, $c_2=0.0374083$, $c_3=0.0320283$,

$c_4=0.0077381$.

Thus, the approximate solution is

$$u(x) = (c_0, c_1, c_2, c_3, c_4)\begin{pmatrix} 1 \\ 2x - 1 \\ 8x^2 - 8x + 1 \\ 32x^3 - 48x^2 + 18x - 1 \\ 128x^4 - 256x^3 + 160x^2 - 32x + 1 \end{pmatrix}$$

$$= 0.990475255x^4 - 0.9560436124x^3 - 1.83671 \times 10^{-41}x.$$

Example 6.2.

consider the following FNFDE from [6]

$$u^{\frac{1}{2}}(t) = -u(t) + \frac{1}{4}u\left(\frac{t}{3}\right) + \frac{1}{3}u^{\frac{1}{2}}\left(\frac{t}{3}\right) + g(t), \quad u(0) = 1, t \in [0,5]$$

Where

$$g(t) = \frac{1}{\Gamma(\frac{1}{2})}\int_0^t (t-s)^{-\frac{1}{2}}e^s ds + e^t - \frac{1}{4}e^{\frac{t}{3}} - \frac{1}{3\Gamma(\frac{1}{2})}\int_0^t (t-s)^{-\frac{1}{2}}e^{\frac{s}{3}} ds.$$

And the exact solution is $u(t) = e^t$.

We apply the suggested method with m=16, 29 and the results are given in Table1.

Table1.The results of the second example.

t	m=16	m=29	Exact solution
0.5	1.4763538174	1.6190417627	1.6487212707
1	3.8801058135	2.7093305549	2.7182818284
1.5	5.7607327414	4.4183810375	4.4816890703
2	8.1454860602	7.309147520	7.3890560989
2.5	11.2503335565	12.219543095	12.1824939607
3	20.2153557325	20.1032043662	20.0855369231
3.5	33.9253810689	33.2483865568	33.1154519586
4	56.6489494633	54.3503822420	54.5981500331
4.5	90.8034308916	89.9912867646	90.0171313005
5	149.1501834012	148.268508213	148.4131591025

7. CONCLUSION

In this paper, we presented an operational matrix method based on shifted Chebyshev tau method for solving the Fractional Neutral Functional-Differential equations. The fractional derivatives are described in the Caputo sense. This method reduced the FNFDEs to a system of linear algebraic equations, which greatly simplifies the problem.

The solution obtained using the suggested method shows that this approach solves the known problem from Bhrawy and Alhamdi [6] effectively.

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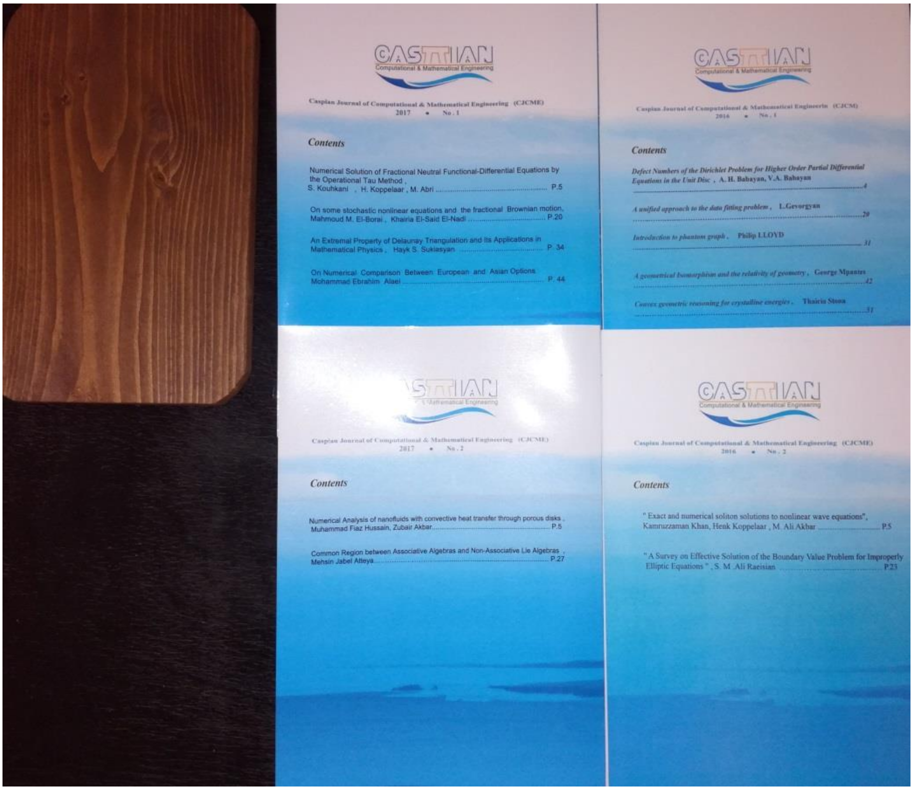

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